LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2015

MT 3875 - MATHEMATICAL FINANCE MODELS

Date : 14/11/2015 Dept. No. Max. : 100 Marks

Time : 09:00-12:00

Answer **ALL** Questions. All question carry equal marks.

1. (a) (i) Explain Geometric Brownian motion.

OR

(ii) Briefly explain doubling rule with an example. (5)

(b) (i) State and Prove Arbitrage theorem.

OR

(ii) An individual who plans to retire in 20 years has decided to put an amount A in the bank at the beginning of each of the next 240 months, after which she will withdraw Rs. 1000 at the beginning of each of the following 360 months. Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be? (15)

2. (a) (i) Prove that the No arbitrage option cost C is decreasing in Strike price K.

OR

(ii) Explain delta edge arbitrage strategy. (5)

(b) (i) Derive the No arbitrage option cost using the Black Scholes formula. (15)

OR

(ii) For two investments, first of which costs the fixed amount C_1 and the second fixed price amount C_2. If the present value from the first investment is always identical to that of the second investment, then either $C_1 = C_2$ or there is an arbitrage. (15)

3. (a) (i)Explain in detail, the conditional value at risk.

OR

(ii) Suppose an investor with capital x can invest any amount between 0 & x ; if y is invested, then y is either won or lost, with respective probabilities *p* &1-*p*. If *p*>1/2, how much should be invested by an investor having a log utility function? (5)

(b) (i) Assuming a General Distribution for the size of a jump, prove that ,

No – arbitrage cost =E[C(s_t, J(t),K, σ, r)] ≥C(s, t, K, σ, r) No arbitrage option cost=

$$C(s,t,K,\pm,r) + s_t^2[e^{-ªt(1-E[J^2])} - e^{-2ªt(1-E[J])}]\frac{1}{2s\pm\sqrt{2\prod t}}e^{-w^2/2}$$

OR

(ii) Prove that in call options on dividend paying securities, for each share owned, a fixed amount D is to be paid at time t_d. (15)

4. (a) (i) Explain Rate of Return with single period Geometric Brownian motion.

OR

(ii) Derive the value of $_i$ in capital assets pricing model. (5)

(b) (i) Estimate the volatility parameter when the collection prices follow Geometric Brownian motion using Opening and Closing data.

OR

(ii) Given three investment projects with the following return functions i) $f_1(x) = \frac{10x}{1+x}, x = 0,1,...$ ii) $f_2(x) = \sqrt{x}, x = 0,1,...$ iii) $f_3(x) = 10(1-e^{-x}), x = 0,1,...$. When we will yield maximum return for we have 5 to invest. (15)

5. (a)(i) Explain the Gambling model with Unknown Win Probabilities.

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OR

(ii) Explain barrier call option with a specified strike price. (5)

(b) (i) Derive the pricing Exotic options by simulation.

OR

(ii) Derive the option cost for options with Nonlinear payoffs. (15)

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